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THE RESONANCE PROBE – A TOOL FOR IONOSPHERIC AND SPACE RESEARCH

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FOREWORD

This interim technical report was prepared by the Microwave Laboratory, W. W. Hansen Laboratories of Physics, Stanford University, Stanford, California, on Contract AF33(616)-8121 for the Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force. The research reported herein was accomplished on Task 7073-03, "Plasma Interactions" of Project 7073, "Research on Plasma Dynamics" under the technical cognizance of Mr. Walter Friz of the Plasma Physics Research Laboratory of ARL.

THE RESONANCE PROBE – A TOOL FOR IONOSPHERIC AND SPACE RESEARCH*

by

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ABSTRACT

Experiments by Yeung and Sayers [1957] on the rf impedance between two probes immersed in a plasma, and those of Takayama and his colleagues [1960] on the incremental dc characteristics of an rf modulated probe, have suggested that resonance effects occur at the local electron plasma frequency, ω_p , and can be interpreted to obtain a direct measurement of electron density, free from many of the errors to which conventional Langmuir probe techniques are prone. Diagnostic techniques based on these experiments have already been applied in ionospheric research. It is shown here that the two resonance effects are strongly related, but that the resonant frequency occurs below ω_p . The extremely complicated theory which describes the resonance probe behavior accurately can be simplified to yield a model which gives predictions agreeing well with experiment. This model is developed and applied to several different probe geometries of interest in ionospheric and space studies. Additional rf techniques are suggested which may prove superior to incremental dc observations for measurement and data coding for transmission to terrestrial stations. An assessment is made of the various regions of applicability of the resonance probe in space, and it is concluded that a detailed theory of the effects of magnetic field on the resonance is required to increase its usefulness for ionospheric probing.

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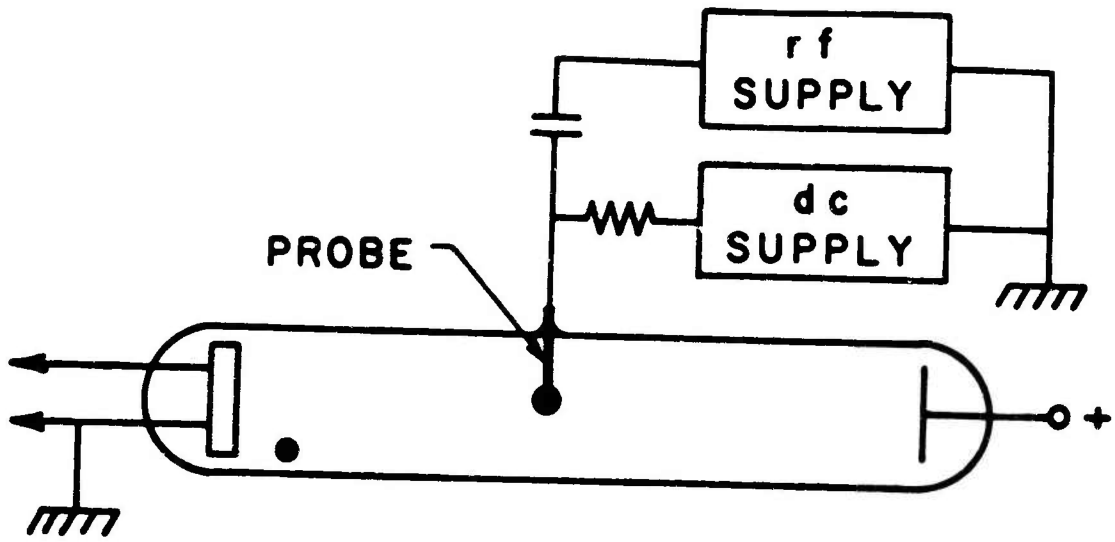
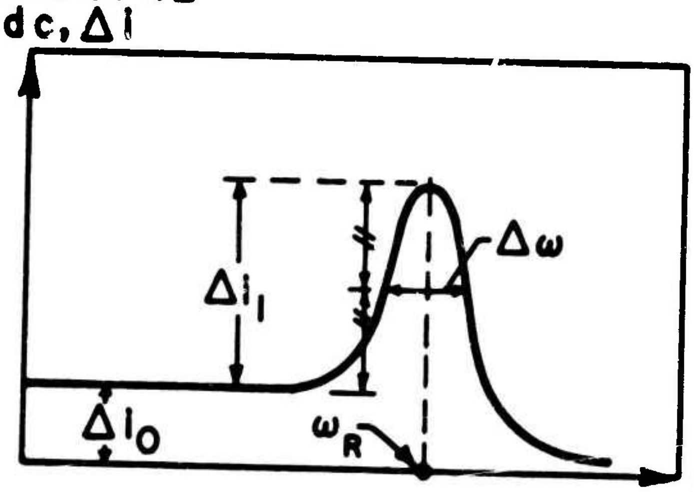
I. INTRODUCTION

In 1957, Yeung and Sayers reported the results of an experiment in which the rf impedance between two electrodes immersed in a plasma was measured. It was found that an impedance minimum occurred at a frequency, ω_R , which probe measurements of electron density led the authors to believe to be the local electron plasma frequency, ω_p . A rather different experiment, which may at first sight appear to be unrelated, was carried out three years later by Takayama and his colleagues [Takayama, et al.; Miyasaki, et al., 1960]. These workers studied the variation with frequency of the incremental dc component of probe current that appears when a simple Langmuir probe immersed in a plasma is modulated with an rf signal [Crawford, 1963]. Their results are sketched in Fig. 1, and gave rise to the description, "resonance probe." From comparisons with electron density measurements obtained by conventional probe techniques, it was concluded that the resonance occurred at ω_p .

The experiments just described have extremely important implications for laboratory experimentation, and for ionospheric and space research since they suggest novel ways in which electron density could be measured directly, without the necessities of using Langmuir probes, or of knowing the electron temperature. This would be of particular value in ionospheric or space studies where Langmuir probes are susceptible to a wide variety of experimental errors [Loeb, 1960], and also in many common



(a) PLASMA TUBE AND PROBE CIRCUIT



(b) VARIATION OF INCREMENTAL dc WITH FREQUENCY

FIG. 1--Characteristics of the resonance probe.

laboratory situations where the electron density is low, or the electron velocity distribution is grossly non-Maxwellian. Several rf and low frequency techniques have already been suggested to reduce the rrors [Crawford and Mlodnosky, 1964]. The results referred to above on plasma resonance suggest further interesting possibilities.

In 1963, it was pointed out by Levitskii and Shashurin that the agreement between ω_R and ω_p obtained by Yeung and Sayers [1957] was erroneous, and that ω_R actually lies below ω_p. Mayer [1963] and Harp [1963, 1964a,b] came to the same conclusion on the operation of the resonance probe. Furthermore, the detailed analyses that have followed have shown that the two experiments describe different aspects of essentially the same phenomenon [Harp, 1964c; Harp and Crawford, 1964].

Our purposes in this paper are to describe briefly the underlying principle of the resonance; to assess its applicability as a diagnostic technique in ionospheric and space research, and to suggest methods by which it may most conveniently be utilized in such measurements. The plan of the paper is as follows. A simplified model of the resonance, which has been justified elsewhere in considerable detail [Harp, 1964c; Harp and Crawford, 1964], is presented in Section (2) together with references to the experimental evidence substantiating it. It is indicated how the theory may be extended to the case in which collisions are important, and the limits within which the resonant frequency must lie when a static magnetic field is present are assessed. In Section (3), simplified formulae are derived for the resonant frequency for probe geometries of interest in ionospheric and space applications. These are discussed in Section (4) in relation to the numerical values of the experimental parameters to be measured in such applications. Suitable techniques for observing the resonance are suggested, and the paper concludes with a general discussion of the potentialities of the resonance probe in Section (5).

II THE OPERATION OF THE RESONANCE PROBE

To understand the mechanism of the resonance, it is necessary to take account of conditions in the plasma-sheath region surrounding the probe. Here, the electron density is lower than in the body of the plasma, and the impedance at frequencies near ω_p will consequently be capacitive. In the body of the plasma, however, the impedance will be inductive, and the possibility arises of a series resonance occurring at some frequency. This may be more easily understood from consideration of a planar system consisting of two probes with a plasma between them. We can then postulate the equivalent circuit shown in Fig. 2. In practice, there will be no sharp division between plasma and sheath, and the relative permittivities will not be exactly $(1 - \omega_p^2/\omega^2)$ and 1 , as shown in the diagram. The circuit does, however, exhibit the features observed experimentally by Levitskii and Shashurin [1963] and sketched in Fig. 2(c) and (d): there is a parallel resonance at ω_p , independent of the sheath thickness, i.e., independent of probe potential. There is a series resonance at $\omega_R (< \omega_p)$, where ω_R can be obtained from the effective dielectric constant, ϵ_{eff} , of the system. For

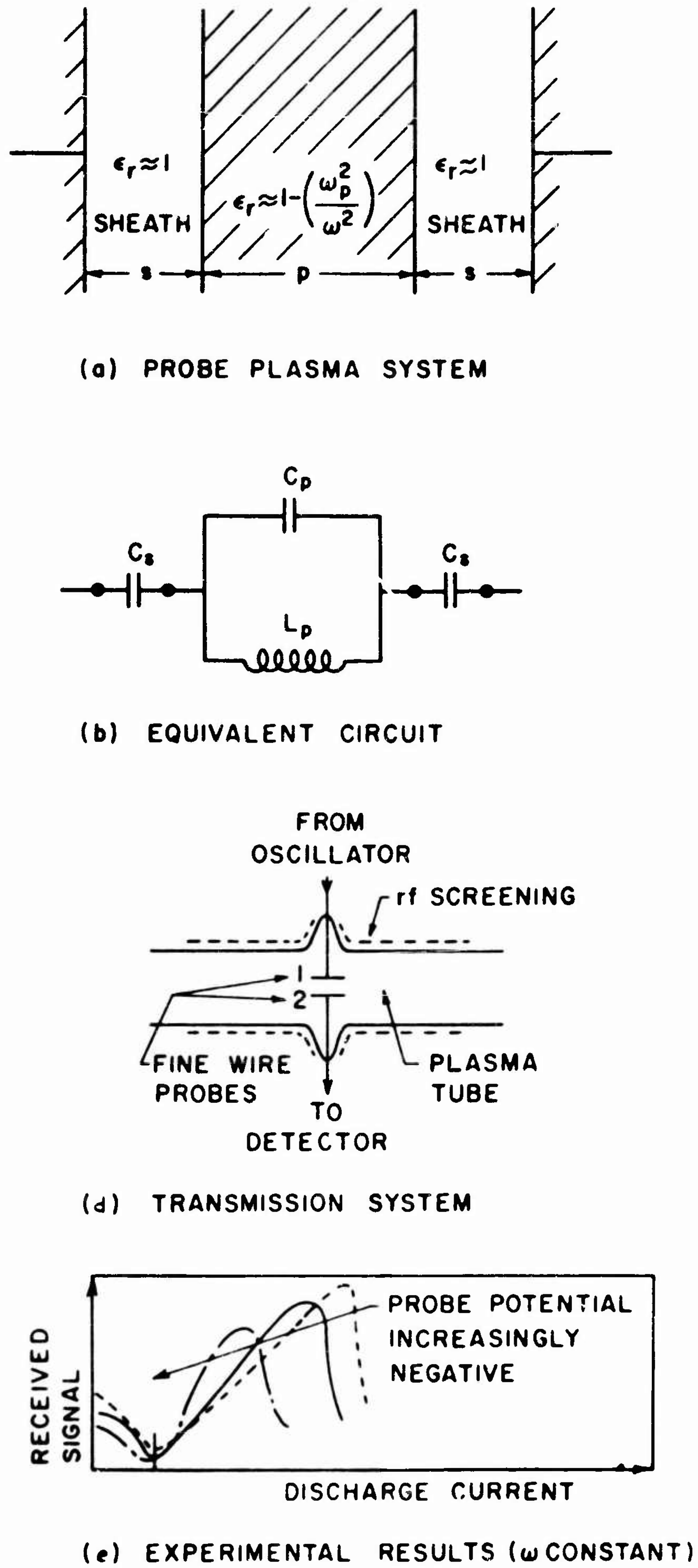


FIG. 2--Probe/plasma system and equivalent circuit.

the planar system this quantity can easily be shown to be

$$\epsilon_{eff} = \epsilon_0 \left\{ \frac{\omega_p^2 - \omega^2}{\dfrac{\omega_p^2}{\left(1 + \dfrac{p}{2s}\right)} - \omega^2} \right\} . \tag{1}$$

The resonant frequency is given by

$$\omega_R = \frac{\omega_p}{\left(1 + \dfrac{p}{2s}\right)^{1/2}} , \tag{2}$$

and should depend on the sheath thickness, as has been shown experimentally [Levitskii and Shashurin, 1963; Peter, et al., 1963; Harp and Crawford, 1964].

We now have to relate this rf phenomenon to the incremental dc peak observed with the resonance probe. This may easily be done from consideration of a phenomenon that has been known for many years. When a Langmuir probe, biased on the electron-repelling part of its characteristic, is modulated with rf there is an increment, Δi_0 , in the dc component of current, i_0 . This is independent of frequency, provided that the frequency is low enough for the probe-sheath conditions to be approximately the same as for dc operation [Crawford, 1963]. For a Maxwellian electron velocity distribution, and a sinusoidal modulating signal, $V_1 \sin \omega t$, the increment is given by

$$\frac{\Delta i_0}{i_0} = I_0 \left(\frac{V_1}{V_e}\right) - 1 , \tag{3}$$

where V_e is the electron temperature, and I_0 is a modified Bessel function of the first kind and zero order. The incremental current increase occurs because the probe characteristic is nonlinear. The rf signal, which is chiefly effective in the sheath region, consequently causes a rectified dc component to flow. In the case of the resonance probe, the rf electric field close to the probe rises near resonance, and causes an additional component of incremental current. Above ω_R, the electric field near to the probe, and hence the incremental dc component, should tend slowly to zero, as observed (see Fig. 1).

The fact that the rf and the dc resonance peaks coincide, was first demonstrated by Harp [1964c], and more recently by Uramoto, et al., [1963a]. The discrepancy between the predicted result that ω_R should not equal ω_p, and the experiments suggesting equality [Takayama, et al., 1960; Miyasaki, et al., 1960; Ikegami, et al., 1963] can probably be attributed to the fact that the only independent check on electron density in the experiments was by Langmuir probes, for which the accuracy was not high in the range of electron densities studied (10^6 - 10^7 electrons/cm^3). The first comprehensive description of the resonance probe showed conclusively, by three independent measurements of electron density consistent to within 20 percent, that ω_R was less than ω_p, and depended on probe potential [Harp, 1963].

The detailed analysis of the probe mechanism is given elsewhere [Harp, 1964c; Harp and Crawford, 1964], and is based on published numerical solutions of the rf electric field distribution near a plane plasma-sheath boundary [Pavkovich and Kino, 1963]. These were obtained by a lengthy numerical integration of the collisionless Boltzmann equation [Pavkovich, 1963]. The results show that an equivalent circuit such as that in Fig. 2(b) can be justified analytically in practical situations, and allow empirical values to be deduced for the constants. These have been checked by direct experiment [Harp and Kino, 1963; Harp, Kino and Pavkovich, 1963]. The way in which the simplified model is determined, for a given geometry, from the infinite plane geometry solutions of Pavkovich and Kino [1963], is described in Section (3), but as an indication of the accuracy with which a simple formula can describe the action of a spherical resonance probe, Fig. 3 is reproduced from the work of Harp and Crawford [1964]. The simplified theoretical formula is

$$\omega_R = \frac{\omega_p}{\left(1 + \frac{R}{k\lambda_D}\right)^{1/2}} , \tag{4}$$

where R is the probe radius, λ_D is the electronic Debye length and k is an empirical constant for the rf sheath thickness expressed in Debye lengths. The results shown in the figure were obtained by three different methods, two of which were rf techniques to be described in Section (5), and the third was by measurement of the frequency at which the peak in the incremental dc occurred. The agreement is very good, and such discrepancy as does exist is substantially due to systematic errors in the guard-ring probe measurements of number density on which ω_p and λ_D were based. These were probably low due to the disturbing influence of the probe on the plasma.

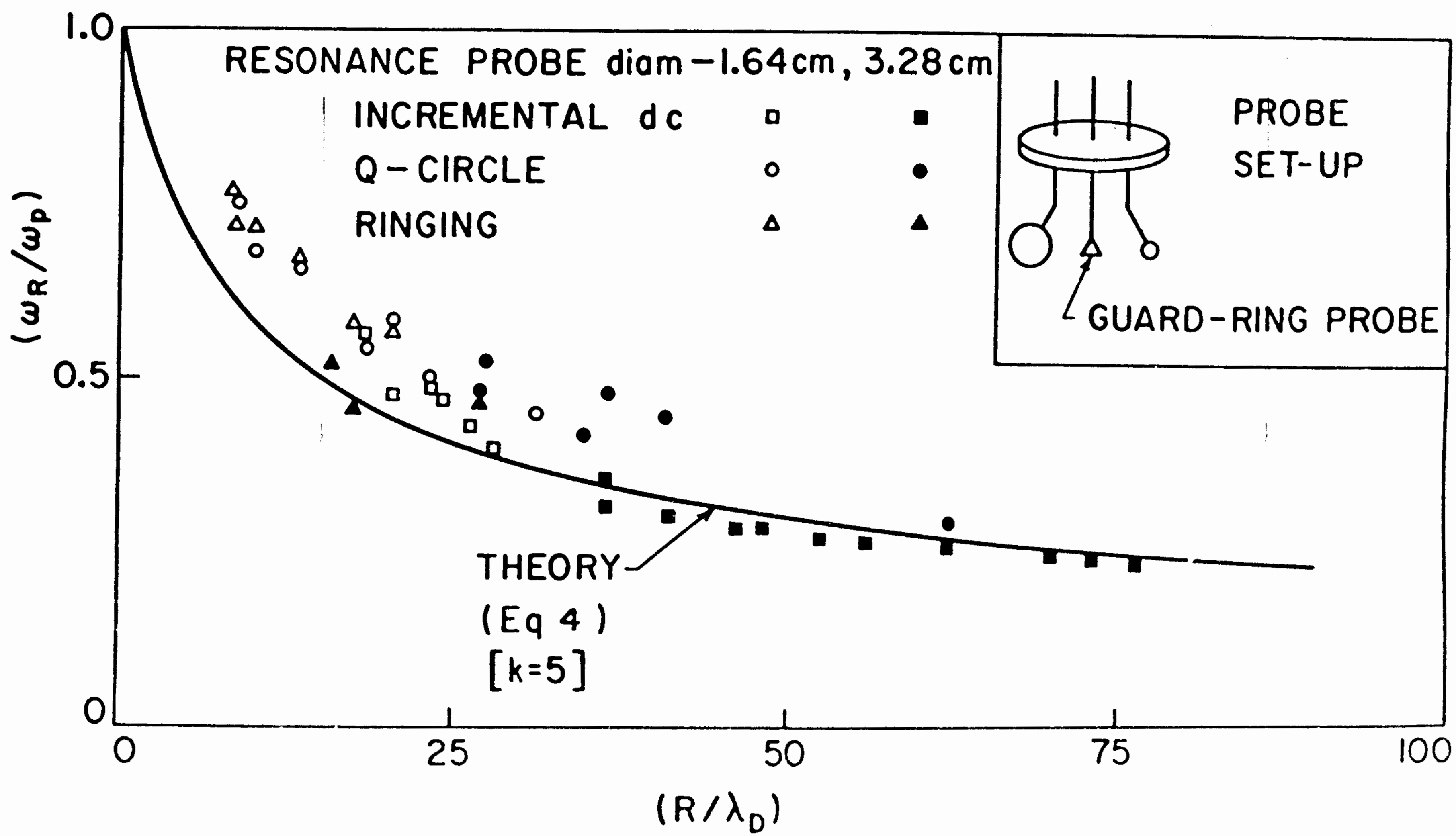
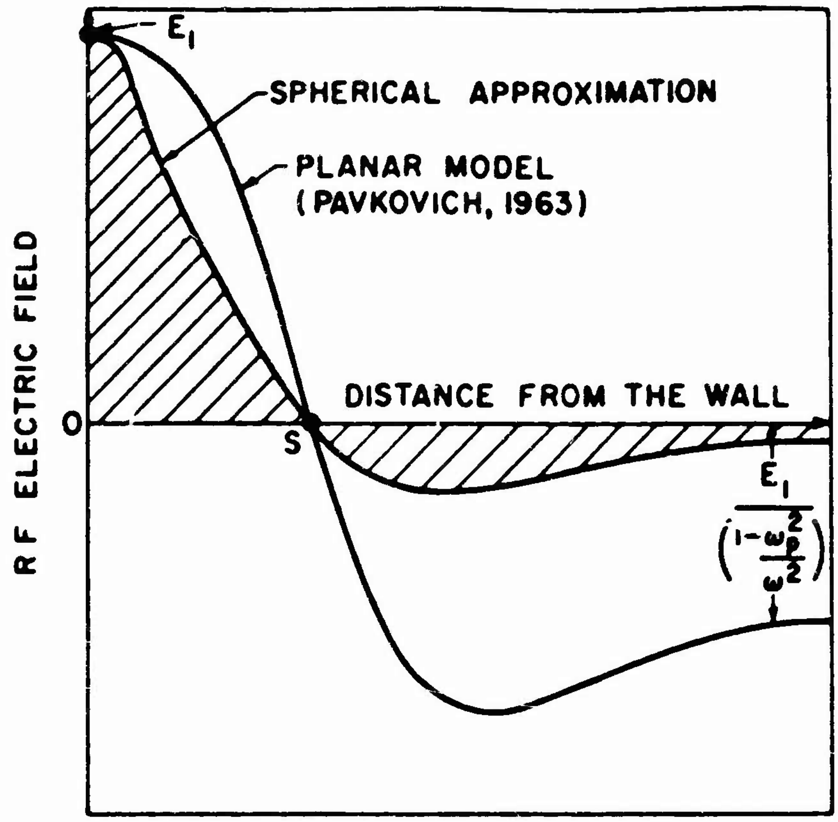


FIG. 3--Probe resonance frequency. Comparison of simplified model and experimental data.

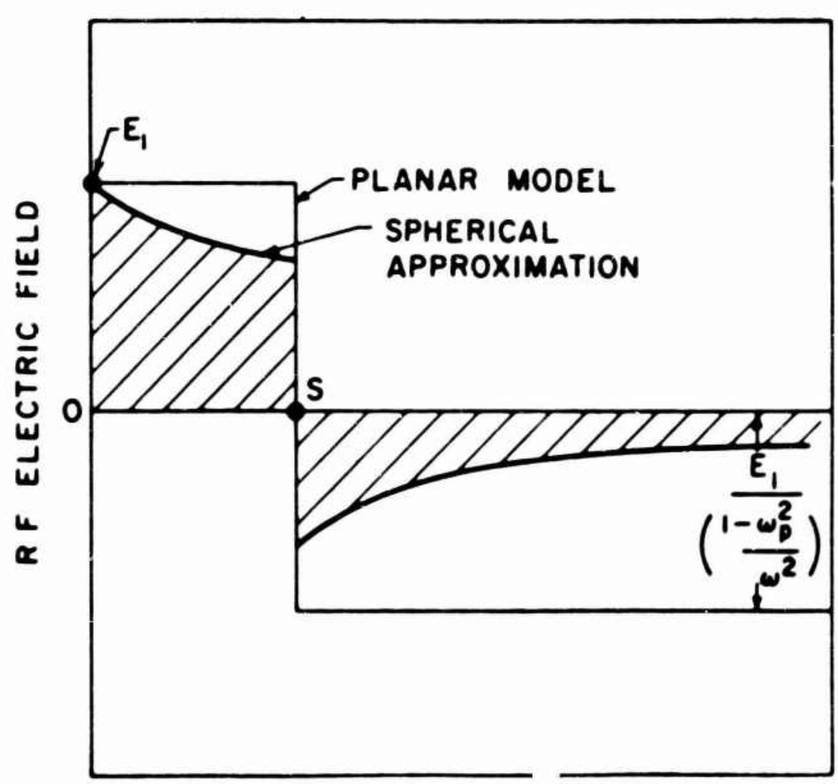
III THE RESONANT FREQUENCY IN SOME PRACTICAL CONFIGURATIONS

A. COLLISIONLESS MODEL WITH NO MAGNETIC FIELD

The way in which a simplified model of resonance probe behavior is built up is indicated in Fig. 4. The detailed theory shows that the rf electric field should have components out of phase and in phase with the applied rf voltage [Pavkovich, 1963]. The in-phase component represents a collisionless damping, similar to Landau damping, and increases rapidly above $\omega_R \approx 0.6\,\omega_p$ to an infinite value as ω_R tends to ω_p. In practice, this implies that probe dimensions must be large compared to the electronic Debye length for a resonance to be observed. When the damping is relatively low, we need consider only the out-of-phase component in obtaining the resonance frequency. A computed curve based on the theory of Pavkovich and Kino is shown in Fig. 4(a) and approximated by a rectangular electric field distribution in Fig. 4(b). To obtain the series resonance frequency we must integrate the electric field outwards from the wall to infinity, and find the frequency for which the resulting potential is zero. In the case of a plane probe bounding a semi-infinite plasma, the potential so obtained is infinite and no series resonance would be observed. However, in a practical case, the probe will be finite, and at distances comparable to the probe dimensions the electric field must fall off proportionately to $(\text{distance})^{-2}$.



(a) APPROXIMATION BASED ON THEORETICAL RESULTS



(b) SIMPLIFIED MODEL OF THE RESONANCE

FIG. 4--Derivation of a simplified model of the resonance.

An approximate electric field profile for a finite plane probe is obtained by multiplying by $(R/r)^2$, as shown in Fig. 4, where R is an empirical length found experimentally for a circular disk to be about equal to the probe radius. For the resonance condition, we find that the shaded area should integrate to zero. Stated mathematically, this implies

$$0 = \int_{R}^{R+s} E_1 \left(\frac{R}{r}\right)^2 dr + \int_{R+s}^{\infty} \frac{E_1}{\left(1 - \frac{\omega_p^2}{\omega^2}\right)} \left(\frac{R}{r}\right)^2 dr \quad , \tag{5}$$

and yields the result quoted in Eq. (4) where $k \approx 5$.

We shall now derive the appropriate formulae for the resonance frequency in various other geometries of interest in laboratory and space research.

1. Isolated Sphere

In the case of a sphere of radius R, the simplified model and the integration are exactly as given in Eq. (5), and yield the result quoted in Eq. (4).

2. Two Spheres

In the case of the dumbbell probe illustrated in Fig. 5(a), or a single sphere at distance a ($\gg R$) from an infinite conducting plane, we can obtain the resonance condition from the equation for the electric

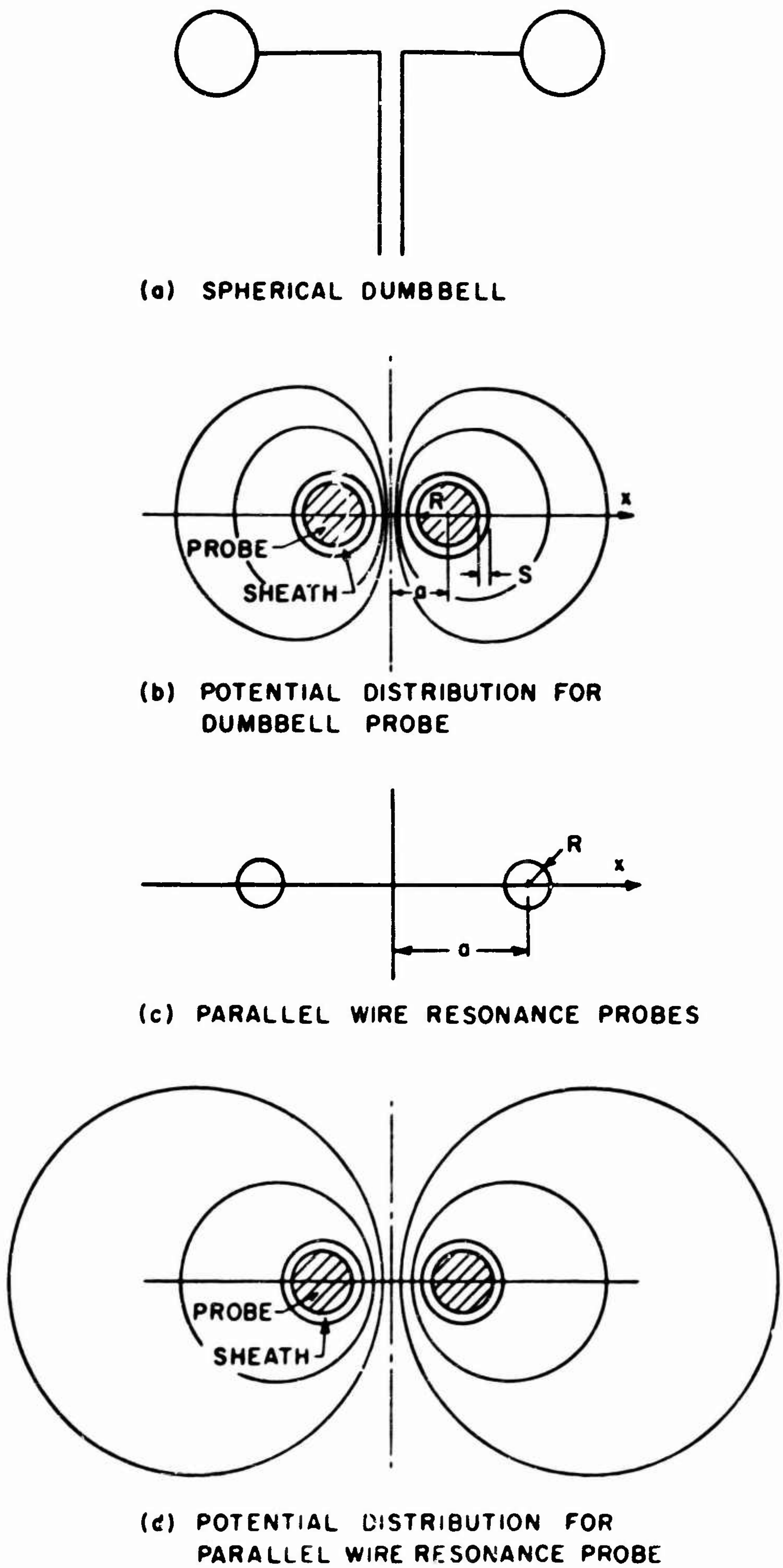


FIG. 5--Some resonance probe geometries of practical interest.

field distribution between two point charges. This assumes, with the introduction of little error, that the rf sheath and the surface of each probe coincide with pairs of slightly nonspherical members of the family of equipotential surfaces. For the electric field on the x-axis we have

$$E_x = \frac{q}{4\pi\epsilon}\left[\frac{1}{(a-x)^2} + \frac{1}{(a+x)^2}\right] . \tag{6}$$

The resonance condition is given, then, by

$$0 = \frac{1}{\left(1 - \frac{\omega_p^2}{\omega^2}\right)} \int_0^{[a-(R+s)]} \left[\frac{1}{(a-x)^2} + \frac{1}{(a+x)^2}\right] dx + \int_{[a-(R+s)]}^{[a-R]} \left[\frac{1}{(a-x)^2} + \frac{1}{(a+x)^2}\right] dx \tag{7}$$

which simplifies to

$$\omega_R^2 = \omega_p^2 \left[1 - \frac{R}{R+s}\left(\frac{1 - \frac{s}{a-R}}{1 - \frac{s}{2a-R}}\right)\right] . \tag{8}$$

Since $a \gg s$ and $(a - R) \gg s$, we have to a close approximation

$$\omega_R^2 = \frac{\omega_p^2}{\left(1 + \frac{R}{s}\right)} . \tag{9}$$

We replace the sheath thickness, s, by $k\lambda_D$ to obtain the result of Eq. (4) again. We remark that Eq. (9) implies that the resonant frequency is not dependent on 'a' to first order.

3. Two Parallel Wires

It may be convenient to use the resonance probe in the form of two parallel wires, or a single wire parallel to a planar surface [see Fig. 5(c)]. The electric field in this case is given for thin wires by the form for the electric field between two point charges,

$$E_x = \frac{q}{2\pi\epsilon}\left[\frac{1}{a - x} + \frac{1}{a + x}\right] \quad , \qquad (10)$$

which yields the resonance condition

$$0 = \frac{1}{\left(1 - \frac{\omega_p^2}{\omega^2}\right)}\int_0^{[a-(R+s)]}\left[\frac{1}{(a - x)} + \frac{1}{(a + x)}\right] dx + \int_{[a-(R+s)]}^{\infty}\left[\frac{1}{a - x} + \frac{1}{a + x}\right] dx \quad .$$

(11)

After integration, this simplifies to

$$\omega_R^2 = \omega_p^2\left[1 - \frac{\ln\left(\frac{2a}{R + s} - 1\right)}{\ln\left(\frac{2a}{R} - 1\right)}\right] \quad . \qquad (12)$$

The implication of this result is that the resonance occurs at rather lower frequencies than for two spheres at the same spacing, and that the spacing has a strong influence on ω_R . This can be understood by considering that the fall-off of the electric field away from the probes varies as $(\text{distance})^{-1}$ in the two-dimensional case. The potential would consequently be infinite at infinite spacing, as in the planar case studied earlier.

B. EFFECTS OF COLLISIONS

So far, we have not considered the damping effect of interparticle collisions on the resonance. This will be additional to the collisionless damping effect already mentioned, and which dominates for probes small compared to a Debye length. Interparticle collisions occurring with frequency ν can be taken into account by modifying the relative permittivity of the plasma to the form

$$\epsilon = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu)} \quad . \tag{13}$$

This separates into real and imaginary parts

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} \quad , \quad \epsilon_i = \frac{\omega_p^2}{\omega^2 + \nu^2}\left(\frac{\nu}{j\omega}\right) \quad . \tag{14}$$

To observe a resonance, it is necessary that $\nu^2 \ll \omega^2$. With this condition, the only modification to be made to Eqs. (4), (9) and (12) to take collisions into account is to modify the left-hand side by replacing ω^2 by $(\omega^2 + \nu^2)$.

In principle, the half-width of the rf series resonance, or of the incremental dc peak, could be used to determine the damping and the collision frequency. Efforts to do so have not yet yielded satisfactory results [Cairns, 1963; Harp and Crawford, 1964].

C. EFFECT OF A STATIC MAGNETIC FIELD

For both laboratory and space studies, use of the resonance probe may be required in situations where a static magnetic field, B , exerts an important influence. So far, the only experimental results that have been obtained under these conditions [Uramoto, et al., 1963b] have shown that, for B small and uniform, ω_R is constant. However, as the magnetic field is raised to strengths such that the electron cyclotron frequency, ω_c , becomes comparable to the zero-field value of ω_R , then ω_R increases and rapidly tends to the limit $\omega_R = \omega_c$. This behavior is illustrated in Fig. 6.

The theory of the series resonance effect for a probe immersed in a magnetic field is a complex problem that has not yet been solved. We may make some tentative estimates, however, of its practical importance by extending the simplified model of Fig. 4. The appropriate expressions for the relative permittivity of a plasma in the directions parallel and

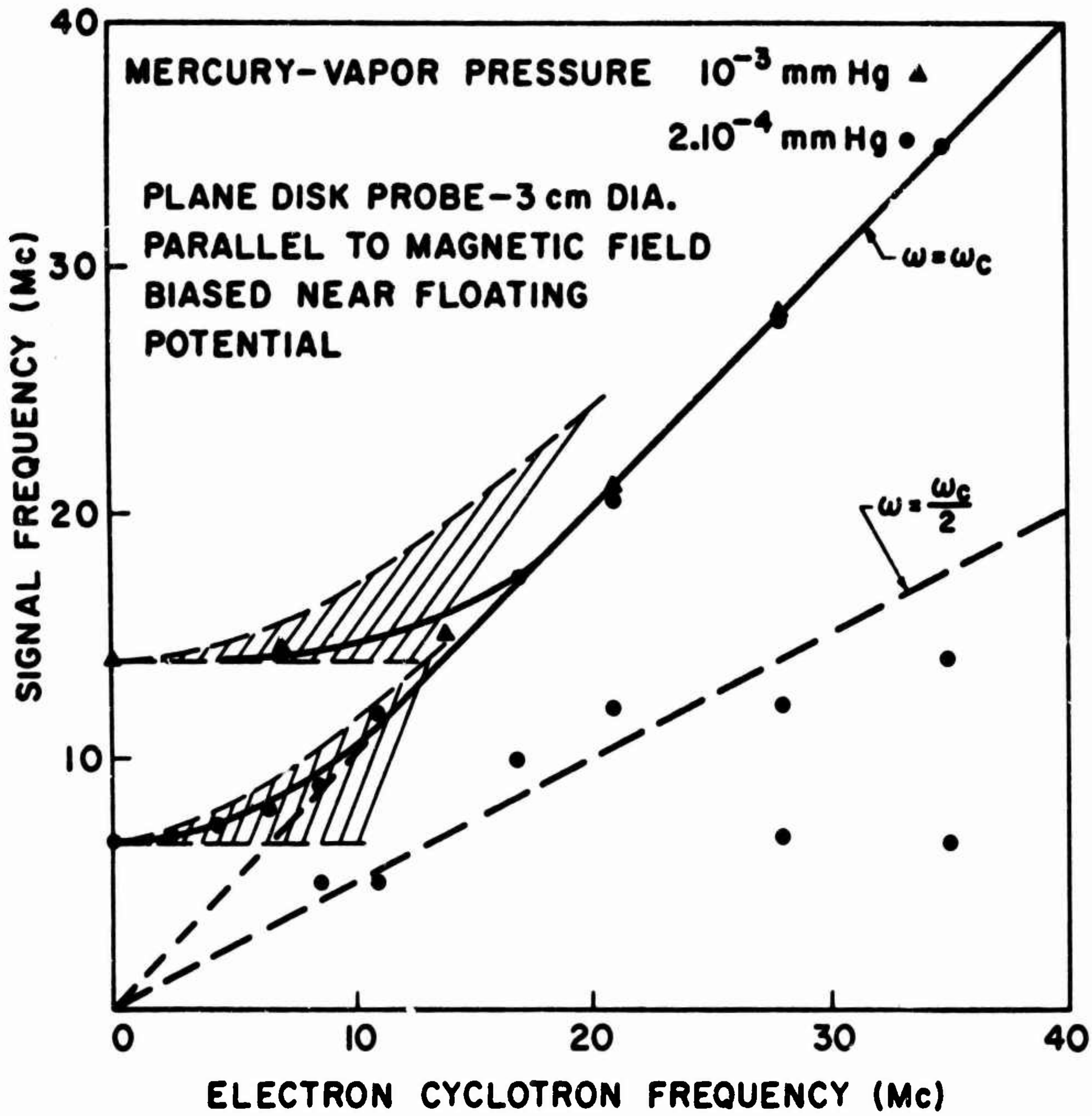


FIG. 6--Behavior of the resonance probe in a magnetic field (after Uramoto, et al., 1963b).

transverse to a static magnetic field are

$$\epsilon_{\parallel} = 1 - \frac{\omega_p^2}{\omega^2} \quad , \quad \epsilon_{\perp} = 1 - \frac{\omega_-^2}{\omega^2 - \omega_c^2} \quad . \tag{15}$$

This suggests that the resonance frequency will lie in a range bounded below by the expression obtained using $\epsilon_{\parallel}$, and above by that obtained using $\epsilon_{\perp}$. The equation giving the upper bound in the spherical case is

$$\omega_R^2 = \frac{\omega_p^2}{\left(1 + \frac{R}{k\lambda_D}\right)} + \omega_c^2 \quad . \tag{16}$$

These considerations define the shaded areas shown in Fig. 6, and within which the experimental data lie.

Experimentally, then, it appears that if ω_p^2 is greater than about $10\ \omega_c^2$, the magnetic field can be neglected. On the other hand, if ω_p^2 is less than $0.1\ \omega_c^2$ it would probably be possible to use the resonance probe to measure magnetic field strength, independent of the local electron density. In the intermediate range $0.1\ \omega_c^2 < \omega_p^2 < 10\ \omega_c^2$ a formula accurately relating (ω_p/ω_c) , (ω_p/ω_R) and (R/λ_D) would be required to interpret the data.

IV PARAMETERS GOVERNING THE USE OF THE RESONANCE PROBE IN IONOSPHERIC AND SPACE APPLICATIONS

We must now examine the parameters to be measured in space in the light of the remarks of the previous section. We can relate them first to the size of the probe necessary to avoid strong collisionless damping. Having ensured that the probe is large enough for a resonance to be observable, we must then examine the value of (ω_p/ω_c) , to determine what the probe will measure, and to decide which theoretical formula is appropriate.

Figure 7 covers the range of electronic Debye lengths and Larmor radii, r_L , likely to be of interest. As the probe radius must be larger than about $(5\lambda_D)$, the probe diameter will be about one order of magnitude larger than the Debye length. Magnetic field effects will be negligible when $r_L \gg \lambda_D$. We see that for the farthest space missions, where the electron temperature may reach 10^5 °K [Chapman, 1959], and the density may be reduced to the order of 1 electron/cm^3, probes exceeding 100 m would be required. This is not entirely out of the question since studies are already under way involving spherical probes of 41 m diameter [project "Echo II," 1964 4A]. For inospheric applications, probes between 1 cm and 1 m in diameter should easily cover the range of electron densities of interest.

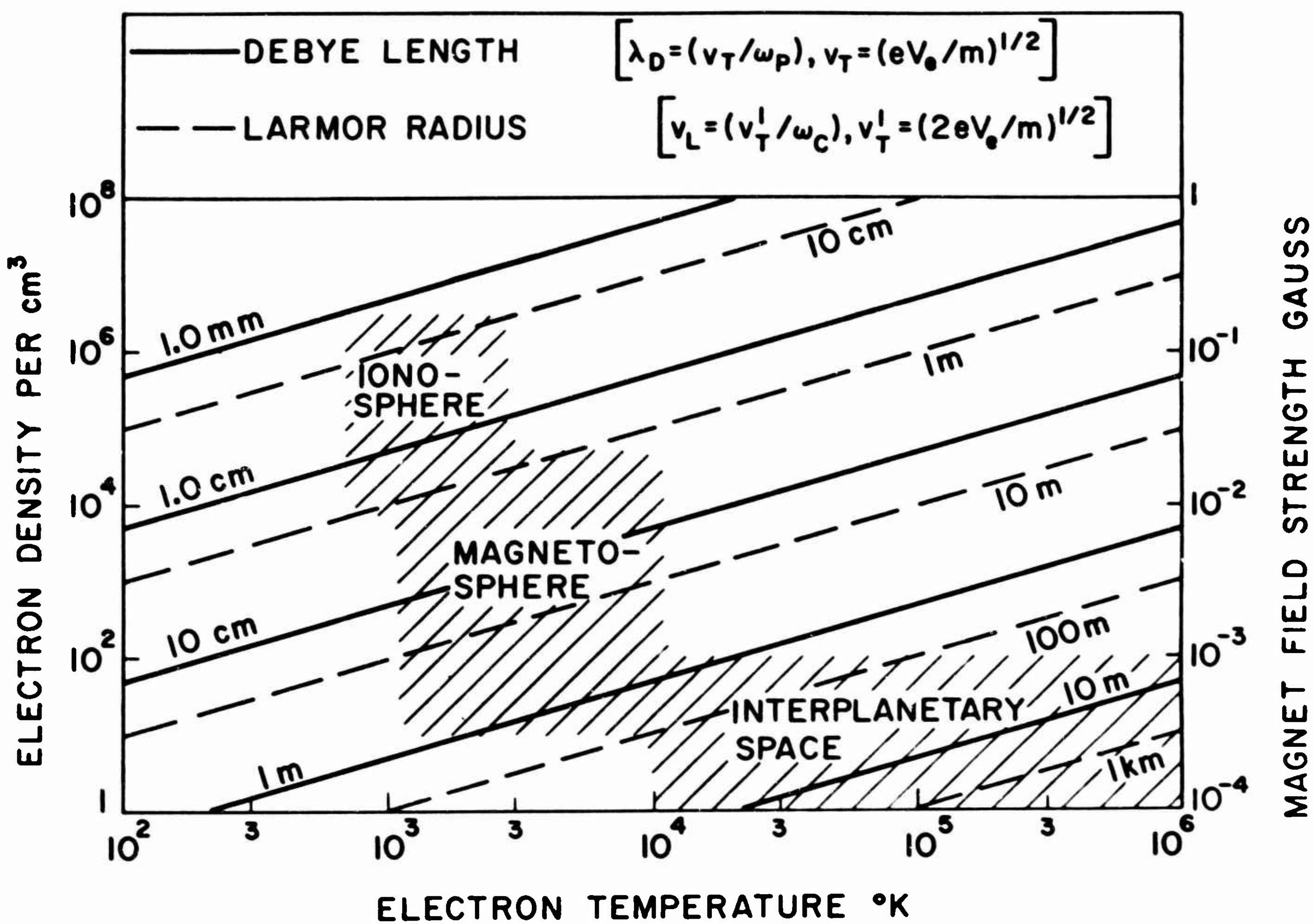


FIG. 7--Probable range of electron density and temperature to be encountered in ionospheric and space applications.

Figure 8 indicates the influence of magnetic field. It will be seen that the resonance probe is suitable for measurements in interplanetary space, and near the edge of the magnetosphere. In the ionosphere, however, a correction formula taking magnetic field into account must be derived if electron densities less than about 10^5 electrons/cm^3 are to be measured accurately, The lower limit seems to be about 10^3 electrons/cm^3. Below this density, the probe would probably be more suitable for magnetic field measurements.

The foregoing considerations suggest that the use of the resonance probe is feasible in all regions of the magnetosphere and interplanetary space. There are, however, additional factors which have to be taken into account. Two of these, which are also effective in reducing the accuracy of simple Langmuir probe measurements, are the effects of particle streaming, and of irradiation. The first of these will have two effects: it will distort the dc plasma sheath, and it will add to the damping of the resonance. The influence of particle streaming will be negligible if the electron drift velocity is small compared to the electron thermal velocity. Irradiation will also change the thickness of the dc plasma sheath. It has been shown empirically [Harp, 1964b] that Eq. (4), which holds only for floating potential, can be modified to take account of probe potential, V_0 , by replacing k by $[(10\ V_0/V_e)^{1/2} - 2]$. This expression is appropriate to mercury-vapor, but could be modified for other gases. It is accurate for V_0 greater than about $(2V_e)$ so that if the probe were normally at floating potential, the resonance frequency would be relatively insensitive to small changes in V_0 caused by irradiation or streaming.

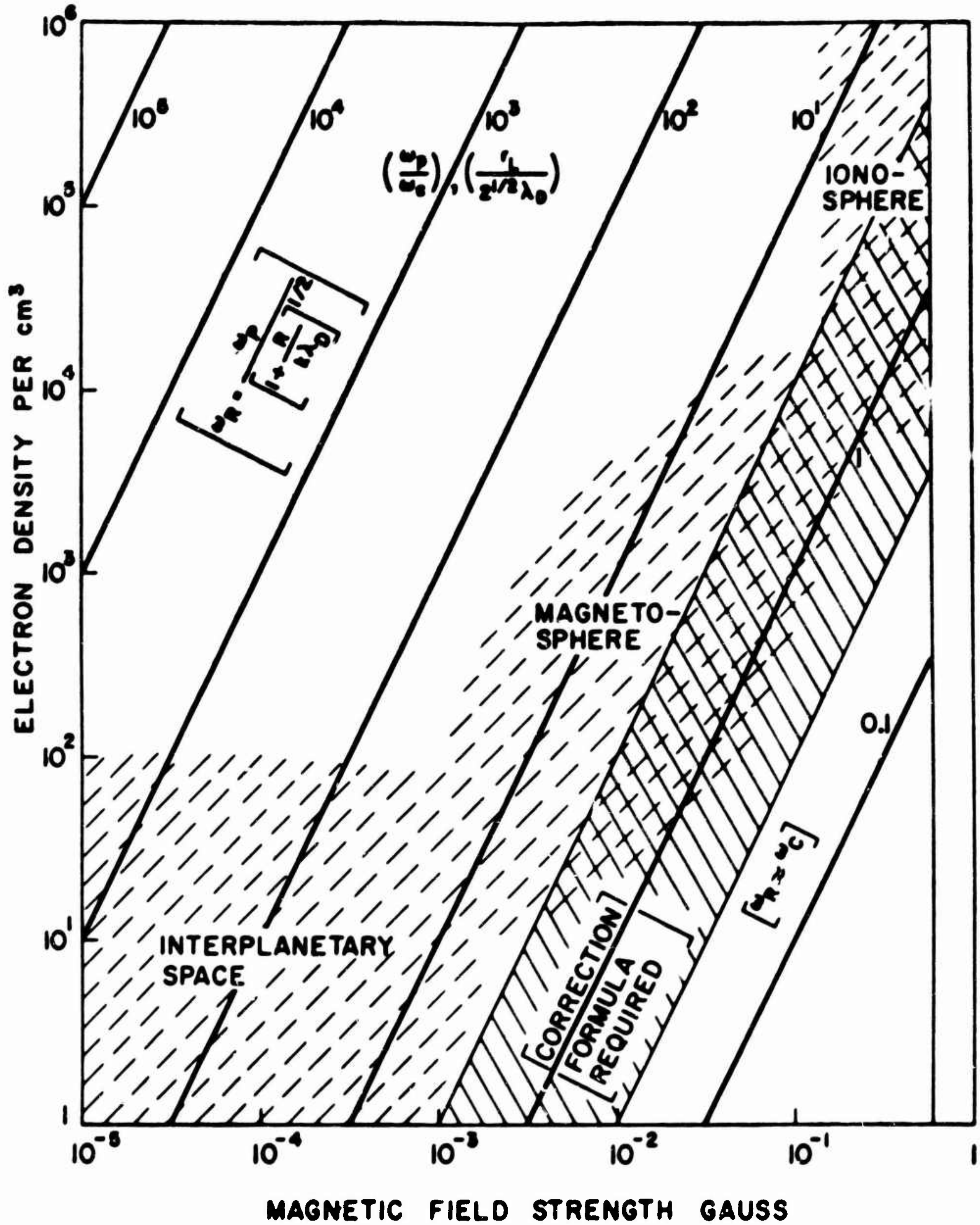


FIG. 8--Ranges of application of the resonance probe.

V DISCUSSION

It is now firmly established from theoretical considerations and experimental measurements that probes immersed in a plasma are capable of exciting a series resonance at frequencies below the local plasma frequency, and that despite its complexity the resonance effect can be summarized in quite simple formulae such as Eqs. (4), (9) and (12). From the considerations of the previous section, it is clear that the resonance probe can constitute a powerful tool for ionospheric and space probing. It will be noted that the resonance frequency can be normalized in the functional form

$$\left(\frac{\omega_R}{\omega_p}\right) = F\left[\left(\frac{R}{\lambda_D}\right) \quad , \quad \left(\frac{a}{\lambda_D}\right) \cdots\right] \quad . \tag{16}$$

This means that measurements can be performed in the laboratory on small-scale models of probes for projected space applications, and that simplified, or empirical, formulae can be checked at moderate cost. Effects such as those of plasma drift could also be conveniently studied in the laboratory.

Determinations of electron density from series resonance measurements will require a subsidiary determination of electron temperature, since this quantity enters via the Debye length. An estimate of electron temperature can be obtained either by using the resonance probe as a conventional Langmuir probe, or by modulating it at a frequency well below ω_R . If the probe is floating, this will cause a shift in potential which can be interpreted to give the electron temperature [Ikegami and Takayama, 1963; Crawford, 1963]. If it is possible to use two spherical probe set-ups of different dimensions, R_1 and R_2 , the Larmor radius can be eliminated to give

$$\omega_p^2 = \frac{R_2 - R_1}{\left(\frac{R_2}{\omega_{R1}^2} - \frac{R_1}{\omega_{R2}^2}\right)} \tag{17}$$

Since the incremental dc resonance is only a manifestation of an rf resonance effect, the measuring techniques are not confined to dc. Two rf methods that have been successfully developed for measuring densities of the order of 10^7 electrons/cm^3 in the laboratory are ringing and Q-circle measurements [Harp and Crawford, 1964]. In the first of these a step-function signal is applied to the probe. This causes a ringing effect to take place at the series resonance frequency, which can then be measured. The Q-circle measurements are slightly more elaborate and require measurement of the rf admittance of the probe-plasma system over a range of frequency. This leads to a circular plot on a Smith chart from which the series resonance can be obtained. It also allows a direct measurement of ω_p to be made as the frequency at which parallel resonance occurs in the plasma and the admittance is a minimum.

As has been pointed out in Section (4), an important region where one would like to employ the resonance probe is in the ionosphere under conditions where $\omega_p \approx \omega_c$. This requires a modified formula for the resonance frequency, and independent knowledge of the local magnetic field. The latter presents no important difficulty. The former is worthy of theoretical study and verification by appropriate laboratory experiments, and would considerably increase the value and applicability of the resonance probe technique.

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